LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 11**

PART A

1. (D) **2.** (D) **3.** (C) **4.** (D) **5.** (B) **6.** (B) **7.** (C) **8.** (C) **9.** (D) **10.** (A) **11.** (C) **12.** (A) **13.** (C) **14.** (C) **15.** (B) **16.** (A) **17.** (A) **18.** (A) **19.** (B) **20.** (A) **21.** (C) **22.** (D) **23.** (D) **24.** (B) **25.** (B) **26.** (A) **27.** (A) **28.** (D) **29.** (C) **30.** (B) **31.** (B) **32.** (B) **33.** (B) **34.** (D) **35.** (B) **36.** (B) **37.** (B) **38.** (D) **39.** (A) **40.** (C) **41.** (A) **42.** (B) **43.** (C) **44.** (B) **45.** (C) **46.** (B) **47.** (D) **48.** (A) **49.** (A) **50.** (C)

PART B

SECTION A

1.

$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Suppose, $\sin^{-1}\frac{1}{2} = \theta$

$\therefore \sin\theta = \frac{1}{2}$

$= \tan^{-1}[2\cos 2\theta]$

$= \tan^{-1}[2(1 - 2\sin^2\theta)]$

$= \tan^{-1}\left[2\left(1 - 2\left(\frac{1}{4}\right)\right)\right] \quad \left(\because \sin\theta = \frac{1}{2}\right)$

$= \tan^{-1}\left[2\left(1 - \frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2 \times \frac{1}{2}\right]$

$= \tan^{-1} 1 = \frac{\pi}{4}$

OR

$= \tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)$

$= \tan^{-1}\left(2\cos\left(2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right)\right)$

$= \tan^{-1}\left(2\cos\left(2\frac{\pi}{6}\right)\right) \quad \left(\because \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$

$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right)$

$= \tan^{-1}\left(2 \cdot \frac{1}{2}\right)$

$= \tan^{-1}(1)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \frac{\pi}{4} \quad \left(\because \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$

2.

Suppose, $\tan^{-1}(\cos x) = \alpha$

$\therefore \cos x = \tan\alpha$

$\therefore 2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\therefore 2\alpha = \tan^{-1}(2\operatorname{cosec} x)$

$\therefore \tan 2\alpha = 2\operatorname{cosec} x$

$\therefore \frac{2\tan\alpha}{1-\tan^2\alpha} = 2\operatorname{cosec} x$

$\therefore \frac{2\cos x}{1-\cos^2 x} = \frac{2}{\sin x} \quad (\because \tan\alpha = \cos x)$

$\therefore 2\frac{\cos x}{\sin^2 x} = \frac{2}{\sin x}$

$\therefore \frac{\cos x}{\sin x} = 1 \quad (\because \sin x \neq 0)$

$\therefore \tan x = 1$

$\therefore x = \frac{\pi}{4}$

Verification :

L.H.S. $= 2\tan^{-1}(\cos x)$

$= 2\tan^{-1}\left(\cos\frac{\pi}{4}\right)$

$= 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \tan^{-1}\frac{1}{\sqrt{2}} + \tan^{-1}\frac{1}{\sqrt{2}}$

$$= tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}}\right)$$

$$= tan^{-1}\left(\frac{2}{\sqrt{2}}\times\frac{2}{1}\right)$$

$$= tan^{-1}\,(2\sqrt{2}\,)$$

R.H.S. $= tan^{-1}\,(2cosecx)$

$$= tan^{-1}\left(2\,cosec\,\frac{\pi}{4}\right)$$

$$= tan^{-1}(2\sqrt{2}\,)$$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ **Solution** $= \left\{\frac{\pi}{4}\right\}$

3.

f is continuous $x = \pi$,

$$\therefore \lim_{x\to\pi^+} f(x) = \lim_{x\to\pi^-} f(x) = f(\pi)$$

$$\therefore \lim_{x\to\pi^+} (cosx) = \lim_{x\to\pi^-} (kx + 1)$$

$$\left(\begin{array}{l}\because x\to\pi^+\\ \Rightarrow x>\pi\\ \Rightarrow f(x)=cos\,x\end{array}\right)\quad\left(\begin{array}{l}\because x\to\pi^-\\ \Rightarrow x<\pi\\ \Rightarrow f(x)=kx+1\end{array}\right)$$

$\therefore \quad cos\pi = k\pi + 1$

$\therefore \quad -1 = k\pi + 1$

$\therefore \quad k\pi = -2$

$\therefore \quad k = \frac{-2}{\pi}$

4.

$$I = \int tan^4x\,dx$$

$$= \int tan^2x \cdot tan^2x\,dx$$

$$= \int tan^2x\,(sec^2x - 1)\,dx$$

$$= \int tan^2x \cdot sec^2x\,dx - \int tan^2x\,dx$$

$$= \int tan^2x\,\frac{d}{dx}\,(tan\,x)\,dx - \int (sec^2x - 1)\,dx$$

$$= \int tan^2x\,\frac{d}{dx}\,(tan\,x)\,dx - \int sec^2x\,dx + \int 1\,dx$$

$$\therefore I = \frac{tan^3x}{3} - tan\,x + x + c$$

5.

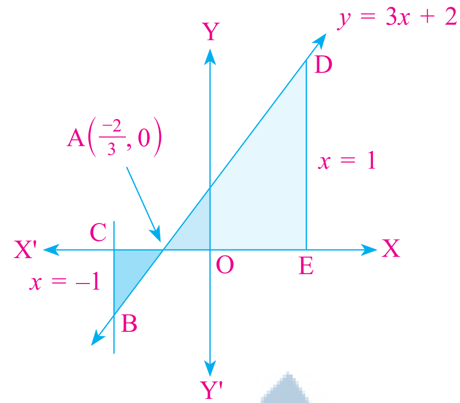
As shown, in the fig. the line $y = 3x + 2$,

meets X-axis at $\left(-\frac{2}{3}, 0\right)$

and its graph lines below

X-axis for $x \in \left(-1, -\frac{2}{3}\right)$

for $x \in \left(-\frac{2}{3}, 1\right)$



The required area

= Area of the region ACBA

+ Area of the required ADEA

$$= \left|\int_{-1}^{-\frac{2}{3}} (3x+2)\,dx\right| + \int_{-\frac{2}{3}}^{1} (3x+2)\,dx$$

$$= \left|\left(\frac{3}{2}x^2+2x\right)_{-1}^{-\frac{2}{3}}\right| + \left(\frac{3}{2}x^2+2x\right)_{-\frac{2}{3}}^{1}$$

$$= \left|\left(\frac{3}{2}\left(\frac{4}{9}\right)-\frac{4}{3}\right)-\left(\frac{3}{2}(1)+2(-1)\right)\right| + \left(\frac{3}{2}(1)+2(1)\right) - \left(\frac{3}{2}\left(\frac{4}{9}\right)+2\left(-\frac{2}{3}\right)\right)$$

$$= \left|\frac{2}{3}-\frac{4}{3}-\frac{3}{2}+2\right| + \frac{3}{2} + 2 - \frac{2}{3} + \frac{4}{3}$$

$$= \left|\frac{-2}{3}-\frac{3}{2}+2\right| + \frac{3}{2} + 2 + \frac{2}{3}$$

$$= \left|\frac{-4-9+12}{6}\right| + \frac{9+12+4}{6}$$

$$= \frac{1}{6} + \frac{25}{6}$$

$$= \frac{26}{6}$$

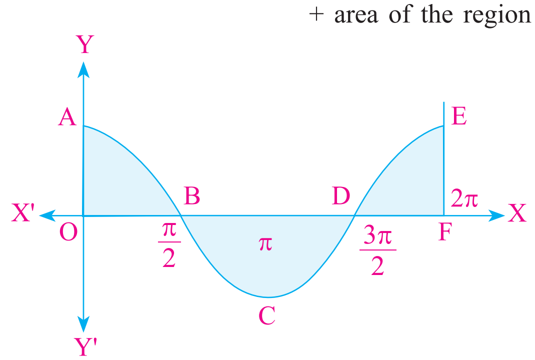
$$= \frac{13}{3} \text{ sq. unit}$$

6.

From the fig. the required area

= area of region OABO + area of region BCDB

+ area of the region DEFD



$\therefore$ Thus, we have the required area.

$$= \int_0^{\frac{\pi}{2}} cosx\, dx + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} cosx\, dx \right| + \int_{\frac{3\pi}{2}}^{2\pi} cosx\, dx$$

$$= (sin\, x)_0^{\frac{\pi}{2}} + \left| (sin\, x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \right| + (sin\, x)_{\frac{3\pi}{2}}^{2\pi}$$

$$= \left(sin\, \frac{\pi}{2} - sin\, 0 \right) + \left| sin\, \frac{3\neq}{2} - sin\, \frac{\neq}{2} \right| + sin\, 2\pi - sin\, \frac{3\pi}{2}$$

$= (1 - 0) + |-1 - 1| + 0 - (-1)$

$= 1 + 2 + 1$

$= 4$ sq. unit

7.

$(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$

$$\therefore\ dy = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}\, dx$$

$\rightarrow$ Integrate both sides :

$$\therefore\ \int dy = \int \frac{(e^x - e^{-x})}{(e^x + e^{-x})}\, dx$$

$$\therefore\ \int dy = \int \frac{\frac{d}{dx}(e^x + e^{-x})}{(e^x + e^{-x})}\, dx$$

$\therefore\ \ y = log\,(e^x + e^{-x}) + c;$

Which is required general solution of given differential equation.

8.

Here,

$$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b}$$

$$= |\vec{a}|^2 - 2(\vec{a} \cdot \vec{b}) + |\vec{b}|^2$$

$$= (2)^2 - 2(4) + (3)^2$$

$$= 5$$

$\therefore\ |\vec{a} - \vec{b}| = \sqrt{5}$

9.

Here, $\dfrac{x-2}{2} = \dfrac{y-1}{5} = \dfrac{z+3}{-3}$

$\therefore\ \ L : \vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(2\hat{i} + 5\hat{j} - 3\hat{k})$

$\vec{b_1} = 2\hat{i} + 5\hat{j} - 3\hat{k}$

and $\dfrac{x+2}{-1} = \dfrac{y-4}{8} = \dfrac{z-5}{4}$

$\therefore\ \ M : \vec{r} = (-2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 8\hat{j} + 4\hat{k})$

$\vec{b_2} = -\hat{i} + 8\hat{j} + 4\hat{k}$

$\vec{b_1} \cdot \vec{b_2} = (2\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 8\hat{j} + 4\hat{k})$

$= -2 + 40 - 12$

$= 26$

$|\vec{b_1}| = \sqrt{4 + 9 + 25}$

$= \sqrt{38}$

$|\vec{b_2}| = \sqrt{1 + 64 + 16}$

$= \sqrt{81}$

$= 9$

If the angle between L and M is α,

$$cos\ \alpha = \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|}$$

$$cos\ \alpha = \frac{|26|}{\sqrt{38}\ 9}$$

$$\therefore\ \ \alpha = cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

Therefore, angle between two line is $cos^{-1}\left(\dfrac{26}{9\sqrt{38}}\right)$.

10.

Line passes through origin,

$\therefore\ \ A(\vec{a}) = (0, 0, 0)$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

The direction of line $\vec{b} = \hat{i}$

($\because$ line is parallel of X-axis)

$\rightarrow$ Now vector equation of lines is,

$\vec{r} = \vec{a} + \lambda\vec{b},\ \lambda \in R$

$\therefore\ \vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda\hat{i}$

Cartesian equation of line,

$$\frac{x-0}{1} = \frac{y}{0} = \frac{z}{0}$$

$$\therefore\ \frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

11.

Event A : first care is black card

$$P(A) = \frac{^{26}C_1}{^{52}C_1} = \frac{1}{2}$$

without replacement,

Event B : Second card is black

$$P(B \mid A) = \frac{^{25}C_1}{^{51}C_1}$$

$$= \frac{25}{51}$$

$\therefore$ Probability of both the cards are black.

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$
$$= \frac{1}{2} \times \frac{25}{51}$$
$$= \frac{25}{102}$$

12.

$S = \{HH, HT, TH, TT\}$

$\therefore n = 4$

(i) E : Tail appears an one coin.

$E : \{HT, TH\}$

Here, $r = 2$

$$\therefore P(E) = \frac{2}{4}$$
$$= \frac{1}{2}$$

F : One coin shows head

$F : \{HT, TH\}$

Here, $r = 2$

$$\therefore P(F) = \frac{1}{2}$$

$$\therefore P(E \cap F) = \frac{1}{2}$$

$$\therefore P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$
$$= \frac{\frac{1}{2}}{\frac{1}{2}}$$
$$= 1$$

SECTION B

13.

Relation S is defined on R

$S = \{(a, b) : a \leq b\}$

Suppose, $(a, a) \in S$

$\therefore$ $a \leq a$ which is true $\forall\ a \in R$

$\therefore$ Assumption is right

$\therefore$ S is reflexive.

Suppose, $(a, b) \in S$

$a \leq b$

$b \leq a$

$(b, a) \notin S$ $\quad$ ($\because$ $a \leq b$ and $b \leq a$ are not same at a time)

$\therefore$ S is not symmetric.

Suppose, $(a, b) \in S$ and $(b, c) \in S$

$\therefore$ $a \leq b$ and $b \leq c$

$\therefore$ $a \leq c$

$\therefore$ $(a, c) \in S$

$\therefore$ S is transitive.

Thus, relation S is reflexive, transitive but not symmetric.

14.

$$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

Now, $$P = \frac{1}{2}(A + A^T)$$
$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right\}$$
$$= \frac{1}{2}\begin{bmatrix} 3+3 & 5+1 \\ 1+5 & -1-1 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$
$$P = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$
$$\therefore P^T = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$
$$\therefore P = P^T$$

$\therefore$ P is Symmetric matrix.

$$Q = \frac{1}{2}(A - A^T)$$
$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right\}$$
$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ -5 & 1 \end{bmatrix}\right\}$$
$$= \frac{1}{2}\begin{bmatrix} 3-3 & 5-1 \\ 1-5 & -1+1 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$
$$Q = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$
$$\therefore Q^T = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$
$$\therefore -Q^T = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$
$$\therefore Q = -Q^T$$

$\therefore$ Q is skew symmetric matrix,

$$\therefore P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 3+0 & 3+2 \\ 3-2 & -1+0 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$

15.

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$\therefore \quad A^3 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

L.H.S. $= A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} + \begin{bmatrix} -36 & 30 & -30 \\ 30 & -36 & 30 \\ -30 & 30 & -36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9+0 & 22-36+18-4 & -21+30-9+0 \\ 21-30+9+0 & -21+30-9+0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= O$

$=$ R.H.S.

$A^3 - 6A^2 + 9A - 4I = O$

Multiply with A^{-1} both sides,

$\therefore \quad (A^3)A^{-1} - 6A^2(A^{-1}) + 9AA^{-1} + 4IA^{-1} = OA^{-1}$

$\therefore \quad A^2 - 6A + 9I - 4A^{-1} = O$

$\therefore \quad 4A^{-1} = A^2 - 6A + 9I$

$$\therefore \quad 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + \begin{bmatrix} -12 & 6 & -6 \\ 6 & -12 & 6 \\ -6 & 6 & -12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\therefore \quad 4A^{-1} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

16.

$$f(x) = \begin{cases} 5 & , \quad x \leqslant 2 \\ ax+b & , \quad 2 < x < 10 \\ 21 & , \quad x \geqslant 10 \end{cases}$$

Here, f is continuous at $x = 2$,

$$\therefore \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = f(2)$$

$$\therefore \lim_{x \to 2^+} (ax + b) = \lim_{x \to 2^-} 5$$

$$\left(\begin{array}{l} \because x \to 2^+ \\ \Rightarrow x > 2 \\ \Rightarrow f(x) = ax + b \end{array}\right) \quad \left(\begin{array}{l} \because x \to 2^- \\ \Rightarrow x < 2 \\ \Rightarrow f(x) = 5 \end{array}\right)$$

$\therefore\ 2a + b = 5$ ... (1)

Now, f is continuous at $x = 10$

$$\therefore \lim_{x \to 10^+} f(x) = \lim_{x \to 10^-} f(x) = f(10)$$

$$\therefore \lim_{x \to 10^+} 21 = \lim_{x \to 10^-} ax + b$$

$$\left(\begin{array}{l} \because x \to 10^+ \\ \Rightarrow x > 10 \\ \Rightarrow f(x) = 21 \end{array}\right) \quad \left(\begin{array}{l} \because x \to 10^- \\ \Rightarrow x < 10 \\ \Rightarrow f(x) = ax + b \end{array}\right)$$

$\therefore\ 21 = 10a + b$

$\therefore\ 10a + b = 21$ .... (2)

Solve the equation (1) and (2),

$$\begin{array}{r} 10a + b = 21 \\ 2a + b = 5 \\ - \quad - \quad - \\ \hline 8a = 16 \\ a = 2 \end{array}$$

Put the value of a in equation (1),

$2(2) + b = 5$

$\therefore\ b = 1$

17.

Take, $I = (-\infty, -1) \cup (1, \infty)$

$I \cap [-1, 1] = \phi$

$$f(x) = x + \frac{1}{x}$$

$\therefore\ f'(x) = 1 - \dfrac{1}{x^2}$ .... (1)

Now, $I = (-\infty, -1) \cup (1, \infty)$

$x \in (-\infty, -1) \cup (1, \infty)$

$\rightarrow \; x^2 > 1$

$\therefore \; \dfrac{1}{x^2} < 1$

$\therefore \; \dfrac{-1}{x^2} > -1$

$\therefore \; 1 - \dfrac{1}{x^2} > 0$

$\therefore \; f'(x) > 0 \qquad (\because$ From equation (1))

$\therefore \; f$ is strictly increasing function on $I = (-\infty, -1) \cup (1, \infty)$.

18.

Here, $\vec{a} \cdot (\vec{b} + \vec{c}) = 0$ and

$\vec{b} \cdot (\vec{a} + \vec{c}) = 0$ and

$\vec{c} \cdot (\vec{a} + \vec{b}) = 0$ is given .... (1)

Now, $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$

$= \vec{a} \cdot \vec{a} + \vec{a} \cdot (\vec{b} + \vec{c})$

$+ \vec{b} \cdot \vec{b} + \vec{b} \cdot (\vec{a} + \vec{c})$

$+ \vec{c} \cdot (\vec{a} + \vec{b}) + \vec{c} \cdot \vec{c}$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$(\because$ From equation (1))

$= 9 + 16 + 25$

$= 50$

Therefore, $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{50}$

$= 5\sqrt{2}$

19.

Two lines are parallel

We have $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$,

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$ and

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Therefore, the distance between lines are,

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$

$$= \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{\sqrt{4+9+36}} \right|$$

$$= \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{7} \text{ unit}$$

20.

$2x + y \geq 3$

$x + 2y \geq 6$

Objective function $Z = x + 2y$

$x \geq 0$

$y \geq 0$

$2x + y = 3$ ... (i)

x	0	$\frac{3}{2}$
y	3	0

$(0, 3)$ ✓

$\left(\frac{3}{2}, 0\right)$ ×

$x + 2y = 6$ ... (ii)

x	0	6
y	3	0

$(0, 3)$ ✓

$(6, 0)$ ×

Solving equation (i) and (ii),

$2x + y = 3$

$2x + 4y = 12$

$- \quad - \quad -$

$3y = 9$

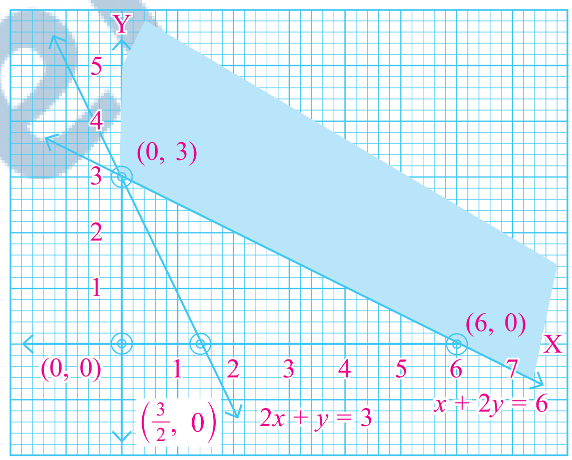
$\therefore \; y = 3$

put $y = 3$ in eq^n (i)

$x + 2(3) = 6$

$\therefore \; x = 6 - 6$

$\therefore \; x = 0$



The shaded region in fig is fasible region determined by the system of costraints which is unbounded The co-ordinates of corner points are (0, 3) and (6, 0)

Corner Point	Corresponding value of $Z = 3x + 5y$
(0, 3)	6
(6, 0)	6

The minimum value of Z occurs at more than two points (0, 3) and (6, 0) joining the line segment.

21.

Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a mediation and yoga centre reduce the risk of heart attack by 30%.

Event E_1 : Meditation and Yoga cource to recover the patient.

Event E_2 : Doctor's prescription of certain drug to reduce heart attack

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

Event A : Person suffers heart attack if patient

Probability of an event that person suffers heart attack if patient followed a course of mediation and Yoga,

$\therefore P(E_1 \mid A) = ?$

$\therefore P(A \mid E_1) =$ Patient has chance of heart attck 70%.

$= 70\% \text{ of } \frac{40}{100}$

$= 0.4 \times \frac{70}{100}$

$= 0.28$

$\therefore P(A \mid E_2) =$ Patient has chance of heart attack 75%.

$= 0.4 \text{ of } 75\%$

$= 0.4 \times \frac{75}{100}$

$= 0.75 \times 0.4$

$= 0.3$

$\therefore P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)$

$= \frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.3$

$= 0.14 + 0.15$

$= 0.29$

$$\therefore P(E_1 \mid A) = \frac{P(A \mid E_1) \cdot P(E_1)}{P(A)}$$

$$= \frac{0.29 \times \frac{1}{2}}{0.29}$$

$$= \frac{14}{29}$$

SECTION C

22.

Total number of Chemistry books = 10 dozen
= 10 × 12
= 120

Total number of Physics books = 8 dozen
= 8 × 12
= 96

Total number of Economics books = 10 dozen
= 10 × 12
= 120

Selling price of Chemistry book is ₹ 80

Selling price of Physics book is ₹ 60

Selling price of Economics book is ₹ 40

Total cost price,

$$= \begin{bmatrix} 120 & 96 & 120 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$= [9600 + 5760 + 4800]$

$= [20160]$

The total amount of the bookshop will receive from selling all the books is ₹ 20,160.

23.

The equation can be represented as matrix form,

$$\therefore \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$\therefore AX = B$

Where, $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$AX = B$

$\therefore X = A^{-1}B$

To find A^{-1},

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 1(1 + 3) + 1(2 + 3) + 1(2 - 1)$

$= 4 + 5 + 1$

$= 10 \neq 0$

$\therefore$ We get unique solution.

For finding *adj* A,

Co-factor of element 1 $A_{11} = (-1)^2 \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix}$
$= 1(1 + 3)$
$= 4$

Co-factor of element –1 $A_{12} = (-1)^3 \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$
$= (-1)(2 + 3)$
$= -5$

Co-factor of element 1 $A_{13} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$
$= 1(2 - 1)$
$= 1$

Co-factor of element 2 $A_{21} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$
$= (-1)(-1 - 1)$
$= 2$

Co-factor of element 1 $A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$
$= 1(1 - 1)$
$= 0$

Co-factor of element –3 $A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$

$= (-1)(1 + 1)$

$= -2$

Co-factor of element 1 $A_{31} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix}$

$= 1(3 - 1)$

$= 2$

Co-factor of element 1 $A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$

$= (-1)(-3 - 2)$

$= 5$

Co-factor of element 1 $A_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$

$= 1(1 + 2)$

$= 3$

$$adj\ A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj\ A = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Solution : $x = 2$, $y = -1$, $z = 1$

24.

$f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$ Lke

Take *log* both side,

$log(f(x)) = log((1 + x)(1 + x^2)(1 + x^4)(1 + x^8))$

$\therefore log(f(x)) = log(1 + x) + log(1 + x^2)$
$+ log(1 + x^4) + log(1 + x^8)$

Now, take differentiation on both sides,

$$\frac{1}{f(x)}\frac{d}{dx}f(x) = \left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

$$\therefore f'(x) = f(x)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

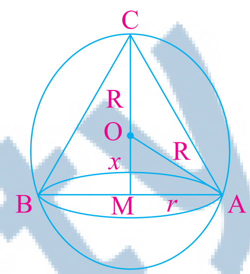
$$\therefore f'(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

$$\therefore f'(1) = (1+1)(1+1)(1+1)(1+1)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$$

$$= (2)(2)(2)(2)\left[\frac{1+2+4+8}{2}\right] = (8)(15)$$

$\therefore f'(1) = 120$

25.



Suppose, radius of base of come is r

$OM = x$

From the fig. height of cone, $h = R + x$

Here, ΔOMA in $R^2 = x^2 + r^2$ .... (1)

$\therefore$ Value of cone, V

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2(x + R)$

$= \frac{1}{3}\pi(R^2 - x^2)(x + R)$ (from equation (1))

$\therefore f(x) = \frac{1}{3}\pi(R^2x + R^3 - x^3 - x^2R)$

$\therefore f'(x) = \frac{1}{3}\pi(R^2 + 0 - 3x^2 - 2xR)$

$\therefore f''(x) = \frac{1}{3}\pi(-6x - 2R)$

$= -\frac{1}{3}\pi(6x + 2R) < 0$ ($\because x > 0$ and $R > 0$)

$\therefore f$ has maximum value.

→ For finding maximum value of cone,

$f'(x) = 0$

$\therefore \frac{\pi}{3}(R^2 - 3x^2 - 2xR) = 0$

$\therefore R^2 - 2xR - 3x^2 = 0$

$\therefore R^2 - 3xR + xR - 3x^2 = 0$

$\therefore R(R - 3x) + x(R - 3x) = 0$

$\therefore (x + R)(R - 3x) = 0$

$\therefore x = -R$ or $x = \frac{R}{3}$

Here, $x > 0$, $x = -R$ is not possible.

$\therefore \quad x = \frac{R}{3}$

Height of cone $(h) = x + R$

$= \frac{R}{3} + R$

$\therefore \quad h = \frac{4R}{3}$

Radius of base of cone, $r^2 = R^2 - x^2$ (from equation (1))

$\rightarrow$ Volume of cone (V) $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(R^2 - x^2)\left(\frac{4R}{3}\right)$

$= \frac{1}{3}\pi\left(R^2 - \frac{R^2}{9}\right)\left(\frac{4R}{3}\right)$

$= \frac{1}{3}\pi\left(\frac{8R^2}{9}\right)\left(\frac{4R}{3}\right)$

$\therefore \quad V = \frac{8}{27} \cdot \frac{4}{3}\pi R^3$

$\therefore$ Volume of cone $= \frac{8}{27} \times$ Volume of sphere.

26.

Method 1 :

$$I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$= \int_0^{\pi} \frac{(\pi - x)}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}dx$$

(By Property (6))

$$= \pi\int_0^{\pi}\frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - \int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$= \pi\int_0^{\pi}\frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - I$$

$$2I = \pi\int_0^{\pi}\frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

Then, $I = \frac{\pi}{2}\int_0^{\pi}\frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$$= \frac{\pi}{2}\cdot 2\int_0^{\frac{\pi}{2}}\frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

(By Property (7))

$$= \pi\left[\int_0^{\frac{\pi}{4}}\frac{dx}{a^2\cos^2 x + b^2\sin^2 x} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{dx}{a^2\cos^2 x + b^2\sin^2 x}\right]$$

$$= \pi\left[\int_0^{\frac{\pi}{4}}\frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\mathrm{cosec}^2 x\,dx}{a^2\cot^2 x + b^2}\right]$$

($\because$ In first integration, each terms divide by $\cos^2 x$ and second integration, each terms divide by $\sin^2 x$)

$$= \pi\left[\int_0^1\frac{dt}{a^2 + b^2t^2} - \int_1^0\frac{du}{a^2u^2 + b^2}\right]$$

($\because$ In first integration, Take $\tan x = t$, in second integration take $\cot x = u$)

$$= \frac{\pi}{ab}\left[\tan^{-1}\frac{bt}{a}\right]_0^1 - \frac{\pi}{ab}\left[\tan^{-1}\frac{au}{b}\right]_1^0$$

$$= \frac{\pi}{ab}\left[\tan^{-1}\frac{b}{a} + \tan^{-1}\frac{a}{b}\right] = \frac{\pi^2}{2ab}$$

Method 2 :

$$I = \int_0^{\pi}\frac{x}{a^2\cos^2 x + b^2\sin^2 x}dx \quad \dots (1)$$

$$= \int_0^{\pi}\frac{(\pi - x)}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}dx$$

$$= \int_0^{\pi}\frac{(\pi - x)}{a^2\cos^2 x + b^2\sin^2 x}dx$$

$$I = \pi\int_0^{\pi}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx - \int_0^{\pi}\frac{x}{a^2\cos^2 x + b^2\sin^2 x}dx$$

$$\therefore I = \pi\int_0^{\pi}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx - I$$

($\because$ From equation (1))

$$\therefore 2I = \pi\int_0^{\pi}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx$$

$$\therefore I = \frac{\pi}{2}\int_0^{\pi}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx$$

$$= \frac{\pi}{2}\left[\int_0^{\frac{\pi}{2}}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx + \int_0^{\frac{\pi}{2}}\frac{1}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}dx\right]$$

$$= \frac{\pi}{2}\left[\int_0^{\frac{\pi}{2}}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx + \int_0^{\frac{\pi}{2}}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx\right]$$

$$= \frac{\pi}{2}\left[2\int_0^{\frac{\pi}{2}}\frac{1}{a^2\cos^2 x + b^2\sin^2 x}dx\right]$$

$$= \pi\int_0^{\frac{\pi}{2}}\frac{\sec^2 x}{a^2 + b^2\tan^2 x}dx$$

($\because$ Divide by each term $\cos^2 x \neq 0$)

Here, Take $tan\ x = t$,

$sec^2 x\ dx = dt$

$x \to 0 \Rightarrow t \to 0$

$x \to \frac{\pi}{2} \Rightarrow t \to \infty$

$$= \pi \int_0^\infty \frac{dt}{a^2 + b^2 t^2}$$

$$= \pi \left[\frac{1}{ab} tan^{-1} \left(\frac{bt}{a} \right) \right]_0^\infty$$

$$= \frac{\pi}{ab} (tan^{-1}(\infty) - tan^{-1}(0))$$

$$= \frac{\pi}{ab} \left(\frac{\pi}{2} - 0 \right)$$

$$= \frac{\pi^2}{2\ ab}$$

27.

$$\therefore \frac{tan^{-1} y - x}{1 + y^2} = \frac{dx}{dy}$$

$$\therefore \frac{dx}{dy} = \frac{tan^{-1} y}{1 + y^2} - \frac{x}{1 + y^2}$$

$$\therefore \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{tan^{-1} y}{1 + y^2}$$

$$\therefore P(y) = \frac{1}{1 + y^2},\ Q(y) = \frac{tan^{-1} y}{1 + y^2}$$

$$\therefore \text{Integrating factor} = e^{\int \frac{1}{1+y^2} dy}$$

$$= e^{tan^{-1} y}$$

→ Thus, the solution of the given differential equation is

$$x e^{tan^{-1} y} = \int \left(\frac{tan^{-1} y}{1 + y^2} \right) e^{tan^{-1} y}\ dy + c$$

Let,

$$I = \int \left(\frac{tan^{-1} y}{1 + y^2} \right) e^{tan^{-1} y}\ dy$$

Put $tan^{-1} y = t$,

$$\left(\frac{1}{1 + y^2} \right) dy = dt$$

$$\therefore I = \int t e^t\ dt$$

$$= t e^t - \int 1 \cdot e^t\ dt$$

$$= t e^t - e^t + c$$

$$= e^t (t - 1) + c$$

$$I = e^{tan^{-1} y} (tan^{-1} y - 1) + c$$

Substituting the value of I in equation (2) we get,

$$x e^{tan^{-1} y} = e^{tan^{-1} y} (tan^{-1} y - 1) + c$$

$$\therefore x = (tan^{-1} y - 1) + c\ e^{-tan^{-1} y}$$

Which is the general solution of the given differential equation.